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Exponential Ratio Type Estimators of Population Mean using Two Auxiliary variables under Non-response

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ABSTRACT

In this paper, we propose exponential ratio type estimators for estimating the population mean of the study variable using information on two auxiliary variables under the situations when certain observations for some sampling units are missing. These missing observations may either be in auxiliary or study variables. The expressions for bias and mean square error of the proposed estimators are obtained up to first order of approximation using simple random sampling without replacement (SRSWOR). Comparison of the proposed exponential ratio type estimators with revised ratio estimators are made both theoretically and through simulation studies. The simulation study showed that proposed estimators are efficient as compared to their competitor estimators.

Keywords: *Auxiliary Variables; Bias; Mean squared error; Non-response.*

INTRODUCTION

Efficiency of an estimator can be increased by using auxiliary information when estimating an unknown population parameter during a sample survey. Most of the times this information is available from the previous studies or Census. Widely used estimators when utilizing auxiliary information are ratio, regression and product estimators. In past few years a lot studies are conducted to estimate the population mean of a variable under study when complete information is available about the variable under study and auxiliary variables (variable which assists in estimation more efficiently) [see Jain (1987), Naik and Gupta (1991), Diana and Perri (2007) and Singh and Agnihotri (2008)).

Currently, most of the researchers are contributing in estimating population parameter in the presence of incomplete information or missing due to various reasons like refusal of respond and unavailability of sampling units when survey is conducted etc. To minimize this bias, it is very important to contact those non-respondents again and try to get complete information through personal interview or any other technique. To

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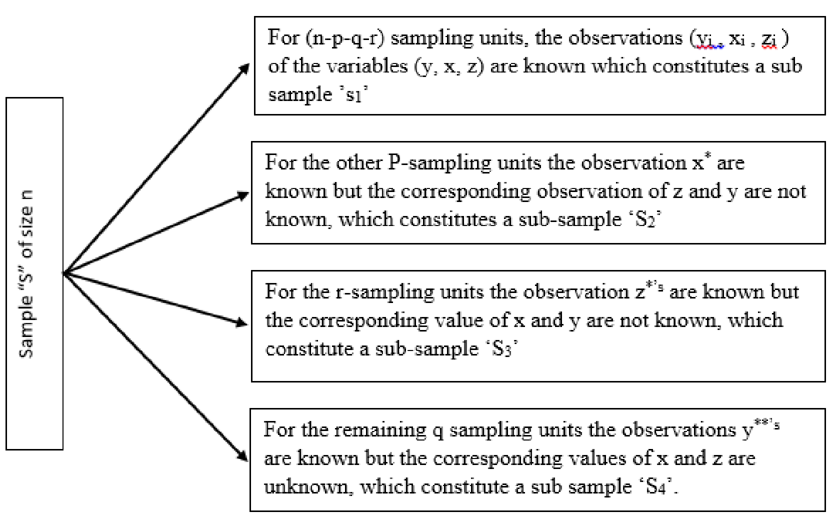
estimate the variable under study while having the problem of non-response most of the work has been done using ratio, product and regression estimators [see Hansen and Hurwitz (1946), Khare and Srivastava (1997), Okafor and Lee (2000), Tabasum and Khan (2004, 2006), Singh and Kumar (2008, 2010)].

When all information about sampling units is available and there is a positive correlation between study and auxiliary variables ratio estimators are used to estimate the unknown population mean of variable under study. But, in case of incomplete information which may occur either in variable under study or auxiliary variable, the conventional way of estimating the unknown parameter cannot be straight forwardly used to analyse the data. It is observed that a huge class of ratio estimators are proposed to estimate population mean under different situations in the presence of non-response either in variable under study or auxiliary variable.

General Notations

Let y,x and z denotes the positive valued variable under consideration and positive valued of the auxiliary variables respectively. Assume the there is a positive correlation among y,x and z . Let (Y_i,X_i,Z_i), i=1,2,3,…….,N denotes the values of the tri-variate (y,x,z) on the i^{ikj} unit of the population of size N. Consider a sample , say "s", of size n is drawn with simple random sampling without replacement (SRSWOR) from this population. Now the problem is to estimate the population mean of the variable under study (y) and assumed that the auxiliary variables has known population mean.

It is assumed that (n-p-q-r) observation of (y,x,z), namely $(y_1,x_1,z_1),(y_2,x_2,z_2)$,. . . .$(y_{n-p-q-r},x_{n-p-q-r},z_{n-p-q-r})$ measured on selected units in the sample are completely available . In addition to these available observation, let $\hat{x}_1$, $\hat{x}_2$,……, $\hat{x}_p$ denotes the available observation of x on these p units in the sample and let $\hat{z}_1,\hat{z}_2,\ldots\ldots,\hat{z}_r$ denotes the available observation of z on these r units in the sample similarly $\hat{y}_1,\hat{y}_2,\ldots\ldots,\hat{y}_q$ denotes the available observation on y on these q units I the sample thus we have the following sub samples of the sample 's'.



To derive the properties of the proposed estimators the following notations are used [Kumar and Kaur (2014)]

$$\bar{x} = \frac{1}{n-p-q-r}\sum_{i\in s_1} x_i\ ,\ \bar{y} = \frac{1}{n-p-q-r}\sum_{i\in s_1} y_i$$

$$\bar{x}^* = \frac{1}{p}\sum_{i\in s_2} x_i^*\ ,\qquad \bar{z}^* = \frac{1}{r}\sum_{i\in s_3} z_i^*$$

$$\bar{y}^{**} = \frac{1}{q}\sum_{i\in s_1\cup s_2} y_i^{**}\ ,\qquad \bar{y}_A = \frac{(n-p-q-r)\bar{y} + q\bar{y}^{**}}{n-p-r}$$

$$\bar{x}_A = \frac{(n-p-q-r)\bar{x} + p\bar{x}^*}{n-q-r}\ ,\qquad \bar{z}_A = \frac{(n-p-q-r)\bar{z} + r\bar{z}^*}{n-p-q}$$

Revised Ratio Estimators of $\bar{Y}$

$$\bar{y}_{r1} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right), \qquad \text{(based on sub-sample } S_1) \qquad (1)$$

$$\bar{y}_{r2} = \bar{y}\left(\frac{\bar{X}}{\bar{x}_A}\right)\left(\frac{\bar{Z}}{\bar{z}}\right), \qquad \text{(based on sub-sample } S_1US_2) \qquad (2)$$

$$\bar{y}_{r3} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}_A}\right), \qquad \text{(based on sub-sample } S_1US_3) \qquad (3)$$

$$\bar{y}_{r4} = \bar{y}_A\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right), \qquad \text{(based on sub-sample } S_1US_4) \qquad (4)$$

$$\bar{y}_{r5} = \bar{y}\left(\frac{\bar{X}}{\bar{x}_A}\right)\left(\frac{\bar{Z}}{\bar{z}_A}\right), \qquad \text{(based on sub-sample } S_1US_2US_3) \qquad (5)$$

$$\bar{y}_{r6} = \bar{y}_A\left(\frac{\bar{X}}{\bar{x}_A}\right)\left(\frac{\bar{Z}}{\bar{z}}\right), \qquad \text{(based on sub-sample } S_1US_2US_4) \qquad (6)$$

$$\bar{y}_{r7} = \bar{y}_A\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}_A}\right), \qquad \text{(based on sub-sample } S_1US_3US_4) \qquad (7)$$

$$\bar{y}_{r8} = \bar{y}_A\left(\frac{\bar{X}}{\bar{x}_A}\right)\left(\frac{\bar{Z}}{\bar{z}_A}\right), \qquad \text{(based on whole sample S)} \qquad (8)$$

To derive the biases and mean square errors of the revised estimators we proceed as follows. Let's take:

$$U=\frac{\bar{x}}{\bar{X}}-1, W=\frac{\bar{z}}{\bar{Z}}-1\ , \quad U^{*}=\frac{\bar{x}^{*}}{\bar{X}}-1\ , \ V^{**}=\frac{\bar{y}^{**}}{\bar{Y}}-1$$

$$V=\frac{\bar{y}}{\bar{Y}}-1\ , \quad W^{*}=\frac{\bar{z}^{*}}{\bar{Z}}-1$$

Under SRSWOR, we have the following expectation

$$E(U)=E(V)=E(W)=E(U^{*})=E(W^{*})=E(V^{**})=0$$

$$E(UU^{*})=E(WW^{*})=E(VV^{**})=E(UW^{*})=E(UV^{**})=E(W^{*}V^{**})=E(U^{*}V^{**})=E(U^{*}W^{*})=0$$

$$E(U^{2})=f_{p+q+r}C_x^2\ , \quad E(V^{2})=f_{p+q+r}C_y^2\ , \ E(W^{2})=f_{p+q+r}C_z^2$$

$$E(U^{*2})=(\frac{1}{p}-\frac{1}{N})C_x^2\ , \ E(W^{*2})=(\frac{1}{r}-\frac{1}{N})C_z^2\ , \quad E(V^{**2})=(\frac{1}{q}-\frac{1}{N})C_y^2$$

$$E(UV)=f_{p+q}\rho_{xy}C_xC_y\ , \ E(UW)=f_{p+r}\rho_{xz}C_xC_z\ , \ E(VW)=f_{q+r}\rho_{yz}C_yC_z$$

We can write the following as in the form

$$\bar{x}=(U+1)\bar{X}\ , \quad \bar{y}=(V+1)\bar{Y}\ , \ \bar{z}=(W+1)\bar{Z}$$

$$\bar{x}^{*}=(U^{*}+1)\bar{X}\ , \quad \bar{y}^{**}=(V^{**}+1)\bar{Y}\ , \ \bar{z}^{*}=(W^{*}+1)\bar{Z}$$

$$C_x^2=\frac{1}{(N-1)\bar{X}^2}\sum_{i=1}^{N}(X_i-\bar{X})^2\ , \qquad C_y^2=\frac{1}{(N-1)\bar{Y}^2}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$$

$$C_z^2=\frac{1}{(N-1)\bar{Z}^2}\sum_{i=1}^{N}(Z_i-\bar{Z})^2 \quad \rho=Corr(y,x)=\frac{1}{(N-1)\bar{X}\bar{Y}C_xC_y}\sum_{i=1}^{N}(X_i-\bar{X})(Y_i-\bar{Y})$$

$$\rho=Corr(y,z)=\frac{1}{(N-1)\bar{Z}\bar{Y}C_zC_y}\sum_{i=1}^{N}(Z_i-\bar{Z})(Y_i-\bar{Y})$$

$$\rho=Corr(y,z)=\frac{1}{(N-1)\bar{Z}\bar{X}C_zC_y}\sum_{i=1}^{N}(Z_i-\bar{Z})(X_i-\bar{X})$$

Biases and Mean Square Errors of the Revised Estimators

$$bias\left(\bar{y}_{r1}\right)\approx\bar{Y}\left(f_{p+q+r}C_x^2+f_{p+q+r}C_z^2-f_{p+q}\rho_{xy}C_xC_y-f_{q+r}\rho_{yz}C_yC_z+f_{p+r}\rho_{xz}C_xC_z\right). \quad (9)$$

$$MSE\left(\bar{y}_{r1}\right) \approx \bar{Y}^2 \begin{pmatrix} f_{p+q+r}C_x^2 + f_{p+q+r}C_z^2 + f_{p+q+r}C_y^2 - 2f_{p+q}\rho_{xy}C_xC_y \\ -2f_{q+r}\rho_{yz}C_yC_z + 2f_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{10}$$

$$bias\left(\bar{y}_{r2}\right) \approx \bar{Y} \begin{pmatrix} A^2 f_{p+q+r}C_x^2 + \lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + f_{p+q+r}C_z^2 \\ -Af_{p+q}\rho_{xy}C_xC_y - f_{q+r}\rho_{yz}C_yC_z + Af_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{11}$$

$$MSE\left(\bar{y}_{r2}\right) \approx \bar{Y}^2 \begin{pmatrix} A^2 f_{p+q+r}C_x^2 + \lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + f_{p+q+r}C_z^2 + f_{p+q+r}C_y^2 \\ -2Af_{p+q}\rho_{xy}C_xC_y - 2f_{q+r}\rho_{yz}C_yC_z + 2Af_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{12}$$

$$bias\left(\bar{y}_{r3}\right) \approx \bar{Y} \begin{pmatrix} F^2 f_{p+q+r}C_z^2 + \lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + f_{p+q+r}C_x^2 \\ +Ff_{p+r}\rho_{xz}C_xC_z - f_{p+q}\rho_{xy}C_xC_y - Ff_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{13}$$

$$MSE\left(\bar{y}_{r3}\right) \approx \bar{Y}^2 \begin{pmatrix} F^2 f_{p+q+r}C_z^2 + \lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + f_{p+q+r}C_x^2 + f_{p+q+r}C_y^2 \\ -2f_{p+q}\rho_{xy}C_xC_y - 2Ff_{q+r}\rho_{yz}C_yC_z + 2Ff_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{14}$$

$$bias\left(\bar{y}_{r4}\right) \approx \bar{Y} \begin{pmatrix} f_{p+q+r}C_x^2 + f_{p+q+r}C_z^2 + f_{p+r}\rho_{xz}C_xC_z \\ -Bf_{p+q}\rho_{xy}C_xC_y - Bf_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{15}$$

$$MSE\left(\bar{y}_{r4}\right) \approx \bar{Y}^2 \begin{pmatrix} B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 + f_{p+q+r}C_x^2 + f_{p+q+r}C_z^2 \\ -2Bf_{p+q}\rho_{xy}C_xC_y - 2Bf_{q+r}\rho_{yz}C_yC_z + 2f_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{16}$$

$$bias\left(\bar{y}_{r5}\right) \approx \bar{Y} \begin{pmatrix} F^2 f_{p+q+r}C_z^2 + \lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + A^2 f_{p+q+r}C_x^2 \\ +\lambda_1^2\left(\frac{1}{r}-\frac{1}{N}\right)C_x^2 + AFf_{p+r}\rho_{xz}C_xC_z \\ -Af_{p+q}\rho_{xy}C_xC_y - Ff_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{17}$$

$$MSE\left(\bar{y}_{r5}\right) \approx \bar{Y}^2 \begin{pmatrix} A^2 f_{p+q+r} C_x^2 + \lambda_1^2 \left(\frac{1}{p} - \frac{1}{N}\right) C_x^2 + F^2 f_{p+q+r} C_z^2 \\ +\lambda_3^2 \left(\frac{1}{r} - \frac{1}{N}\right) C_z^2 + f_{p+q+r} C_y^2 - 2A f_{p+q} \rho_{xy} C_x C_y \\ -2F f_{q+r} \rho_{yz} C_y C_z + 2AF f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \quad (18$$

$$bias\left(\bar{y}_{r6}\right) \approx \bar{Y} \begin{pmatrix} A^2 f_{p+q+r} C_x^2 + \lambda_1^2 \left(\frac{1}{p} - \frac{1}{N}\right) C_x^2 + f_{p+q+r} C_z^2 \\ +A f_{p+r} \rho_{xz} C_x C_z - AB f_{p+q} \rho_{xy} C_x C_y - B f_{q+r} \rho_{yz} C_y C_z \end{pmatrix}. \quad (19)$$

$$MSE\left(\bar{y}_{r6}\right) \approx \bar{Y}^2 \begin{pmatrix} A^2 f_{p+q+r} C_x^2 + \lambda_1^2 \left(\frac{1}{p} - \frac{1}{N}\right) C_x^2 + f_{p+q+r} C_z^2 \\ +B^2 f_{p+q+r} C_y^2 + \lambda_2^2 \left(\frac{1}{q} - \frac{1}{N}\right) C_y^2 - 2AB f_{p+q} \rho_{xy} C_x C_y \\ -2B f_{q+r} \rho_{yz} C_y C_z + 2A f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \quad (20)$$

$$bias\left(\bar{y}_{r7}\right) \approx \bar{Y} \begin{pmatrix} F^2 f_{p+q+r} C_z^2 + \lambda_3^2 \left(\frac{1}{r} - \frac{1}{N}\right) C_z^2 + f_{p+q+r} C_x^2 \\ +F f_{p+r} \rho_{xz} C_x C_z - B f_{p+q} \rho_{xy} C_x C_y - BF f_{q+r} \rho_{yz} C_y C_z \end{pmatrix}. \quad (21)$$

$$MSE\left(\bar{y}_{r7}\right) \approx \bar{Y}^2 \begin{pmatrix} F^2 f_{p+q+r} C_z^2 + \lambda_3^2 \left(\frac{1}{r} - \frac{1}{N}\right) C_z^2 + f_{p+q+r} C_x^2 \\ +B^2 f_{p+q+r} C_y^2 + \lambda_2^2 \left(\frac{1}{q} - \frac{1}{N}\right) C_y^2 - 2B f_{p+q} \rho_{xy} C_x C_y \\ -2BF f_{q+r} \rho_{yz} C_y C_z + 2F f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \quad (22)$$

$$bias\left(\bar{y}_{r8}\right) \approx \bar{Y}\begin{pmatrix} A^2 f_{p+q+r}C_x^2 + \lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + F^2 f_{p+q+r}C_z^2 \\ +\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + ABf_{p+r}\rho_{xz}C_xC_z \\ -AFf_{p+q}\rho_{xy}C_xC_y - BFf_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \qquad (23)$$

$$MSE\left(\bar{y}_{r8}\right) \approx \bar{Y}^2\begin{pmatrix} A^2 f_{p+q+r}C_x^2 + \lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + F^2 f_{p+q+r}C_z^2 \\ +\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 \\ -2AFf_{p+q}\rho_{xy}C_xC_y - 2BFf_{q+r}\rho_{yz}C_yC_z \\ +2ABf_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \qquad (24)$$

Proposed exponential-type ratio estimator for population mean ($\bar{Y}$)

$$\bar{y}_{er1} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right), \quad \text{(based on sub-sample } S_1\text{)} \qquad (25)$$

$$\bar{y}_{er2} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}_A}{\bar{X}+\bar{x}_A}\right)\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right), \quad \text{(based on sub-sample } S_1US_2\text{)} \qquad (26)$$

$$\bar{y}_{er3} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\exp\left(\frac{\bar{Z}-\bar{z}_A}{\bar{Z}+\bar{z}_A}\right), \quad \text{(based on sub-sample } S_1US_3\text{)} \qquad (27)$$

$$\bar{y}_{er4} = \bar{y}_A\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right), \quad \text{(based on sub-sample } S_1US_4\text{)} \qquad (28)$$

$$\bar{y}_{er5} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}_A}{\bar{X}+\bar{x}_A}\right)\exp\left(\frac{\bar{Z}-\bar{z}_A}{\bar{Z}+\bar{z}_A}\right), \quad \text{(based on sub-sample } S_1US_2US_3\text{)} \qquad (29)$$

$$\bar{y}_{er6} = \bar{y}_A\exp\left(\frac{\bar{X}-\bar{x}_A}{\bar{X}+\bar{x}_A}\right)\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right), \quad \text{(based on sub-sample } S_1US_2US_4\text{)} \qquad (30)$$

$$\bar{y}_{er7} = \bar{y}_A \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \exp\left(\frac{\bar{Z} - \bar{z}_A}{\bar{Z} + \bar{z}_A}\right), \qquad \text{(based on sub-sample } S_1US_3US_4\text{)} \qquad (31)$$

$$\bar{y}_{er8} = \bar{y}_A \exp\left(\frac{\bar{X} - \bar{x}_A}{\bar{X} + \bar{x}_A}\right) \exp\left(\frac{\bar{Z} - \bar{z}_A}{\bar{Z} + \bar{z}_A}\right), \qquad \text{(based on whole sample S)} \qquad (32)$$

These proposed exponential type ratio estimators will be compared with the corresponding revised estimators with respect to mean square errors.

Biases and Mean Square Errors of the proposed exponential type ratio Estimators

$$bias\left(\bar{y}_{r1}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8} f_{p+q+r} C_x^2 + \frac{3}{8} f_{p+q+r} C_z^2 - \frac{1}{2} f_{p+q} \rho_{xy} C_x C_y \\ -\frac{1}{2} f_{q+r} \rho_{yz} C_y C_z + \frac{1}{4} f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \qquad (33)$$

$$MSE\left(\bar{y}_{r1}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4} f_{p+q+r} C_x^2 + \frac{1}{4} f_{p+q+r} C_z^2 + f_{p+q+r} C_y^2 \\ - f_{p+q} \rho_{xy} C_x C_y - f_{q+r} \rho_{yz} C_y C_z + \frac{1}{2} f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \qquad (34)$$

$$bias\left(\bar{y}_{r2}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8} A^2 f_{p+q+r} C_x^2 + \frac{3}{8} \lambda_1^2 \left(\frac{1}{p} - \frac{1}{N}\right) C_x^2 + \frac{3}{8} f_{p+q+r} C_z^2 \\ -\frac{1}{2} A f_{p+q} \rho_{xy} C_x C_y - \frac{1}{2} f_{q+r} \rho_{yz} C_y C_z + \frac{1}{4} A f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \qquad (35)$$

$$MSE\left(\bar{y}_{r2}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4} A^2 f_{p+q+r} C_x^2 + \frac{1}{4} \lambda_1^2 \left(\frac{1}{p} - \frac{1}{N}\right) C_x^2 \\ + \frac{1}{4} f_{p+q+r} C_z^2 + f_{p+q+r} C_y^2 - A f_{p+q} \rho_{xy} C_x C_y \\ - f_{q+r} \rho_{yz} C_y C_z + \frac{1}{2} A f_{p+r} \rho_{xz} C_x C_z \end{pmatrix}. \qquad (36)$$

$$bias\left(\bar{y}_{r3}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8}F^2 f_{p+q+r}C_z^2 + \frac{3}{8}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + \frac{3}{8}f_{p+q+r}C_x^2 \\ +\frac{1}{4}Ff_{p+r}\rho_{xz}C_xC_z - \frac{1}{2}f_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}Ff_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{37}$$

$$MSE\left(\bar{y}_{r3}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4}F^2 f_{p+q+r}C_z^2 + \frac{1}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + \frac{1}{4}f_{p+q+r}C_x^2 \\ + f_{p+q+r}C_y^2 - f_{p+q}\rho_{xy}C_xC_y - Ff_{q+r}\rho_{yz}C_yC_z \\ +\frac{1}{2}Ff_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{38}$$

$$bias\left(\bar{y}_{r4}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8}f_{p+q+r}C_x^2 + \frac{3}{8}f_{p+q+r}C_z^2 + \frac{1}{4}f_{p+r}\rho_{xz}C_xC_z \\ -\frac{1}{2}Bf_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}Bf_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{39}$$

$$MSE\left(\bar{y}_{r4}\right) \approx \bar{Y}^2\begin{pmatrix} B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 + \frac{1}{4}f_{p+q+r}C_x^2 + \frac{1}{4}f_{p+q+r}C_z^2 \\ -Bf_{p+q}\rho_{xy}C_xC_y - Bf_{q+r}\rho_{yz}C_yC_z + \frac{1}{2}f_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{40}$$

$$bias\left(\bar{y}_{r5}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8}F^2 f_{p+q+r}C_z^2 + \frac{3}{8}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + \frac{3}{8}A^2 f_{p+q+r}C_x^2 \\ +\frac{3}{8}\lambda_1^2\left(\frac{1}{r}-\frac{1}{N}\right)C_x^2 + \frac{1}{4}AFf_{p+r}\rho_{xz}C_xC_z \\ -\frac{1}{2}Af_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}Ff_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \tag{41}$$

$$MSE\left(\bar{y}_{r5}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4}A^2 f_{p+q+r}C_x^2 + \frac{1}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + \frac{1}{4}F^2 f_{p+q+r}C_z^2 \\ +\frac{1}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + f_{p+q+r}C_y^2 - Af_{p+q}\rho_{xy}C_xC_y \\ -Ff_{q+r}\rho_{yz}C_yC_z + \frac{1}{2}AFf_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \tag{42}$$

$$bias\left(\bar{y}_{r6}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8}A^2 f_{p+q+r}C_x^2 + \frac{3}{8}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 \\ +\frac{3}{8} f_{p+q+r}C_z^2 + \frac{1}{4}Af_{p+r}\rho_{xz}C_xC_z \\ -\frac{1}{2}ABf_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}Bf_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \qquad (43)$$

$$MSE\left(\bar{y}_{r6}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4}A^2 f_{p+q+r}C_x^2 + \frac{1}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 + \frac{1}{4} f_{p+q+r}C_z^2 \\ +B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 - ABf_{p+q}\rho_{xy}C_xC_y \\ -Bf_{q+r}\rho_{yz}C_yC_z + \frac{1}{2}Af_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \qquad (44$$

$$bias\left(\bar{y}_{r7}\right) \approx \bar{Y}\begin{pmatrix} \frac{3}{8}F^2 f_{p+q+r}C_z^2 + \frac{3}{8}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 + \frac{3}{8} f_{p+q+r}C_x^2 \\ +\frac{1}{4}Ff_{p+r}\rho_{xz}C_xC_z - \frac{1}{2}Bf_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}BFf_{q+r}\rho_{yz}C_yC_z \end{pmatrix}. \qquad (45)$$

$$MSE\left(\bar{y}_{r7}\right) \approx \bar{Y}^2\begin{pmatrix} \frac{1}{4}F^2 f_{p+q+r}C_z^2 + \frac{1}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 \\ +\frac{1}{4} f_{p+q+r}C_x^2 + B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 \\ -Bf_{p+q}\rho_{xy}C_xC_y - BFf_{q+r}\rho_{yz}C_yC_z \\ +\frac{1}{2}Ff_{p+r}\rho_{xz}C_xC_z \end{pmatrix}. \qquad (46)$$

$$bias\left(\bar{y}_{r8}\right) \approx \bar{Y}\left(\begin{array}{l} \frac{3}{8}A^2 f_{p+q+r}C_x^2 + \frac{3}{8}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 \\ +\frac{3}{8}F^2 f_{p+q+r}C_z^2 + \frac{3}{8}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 \\ +B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{p}-\frac{1}{N}\right)C_y^2 \\ -\frac{1}{2}AFf_{p+q}\rho_{xy}C_xC_y - \frac{1}{2}BFf_{q+r}\rho_{yz}C_yC_z \\ +\frac{1}{4}ABf_{p+r}\rho_{xz}C_xC_z \end{array}\right). \tag{47}$$

$$MSE\left(\bar{y}_{er8}\right) \approx \bar{Y}^2\left(\begin{array}{l} \frac{1}{4}A^2 f_{p+q+r}C_x^2 + \frac{1}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2 \\ +\frac{1}{4}F^2 f_{p+q+r}C_z^2 + \frac{1}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2 \\ +B^2 f_{p+q+r}C_y^2 + \lambda_2^2\left(\frac{1}{q}-\frac{1}{N}\right)C_y^2 \\ -AFf_{p+q}\rho_{xy}C_xC_y - BFf_{q+r}\rho_{yz}C_yC_z \\ +\frac{1}{2}ABf_{p+r}\rho_{xz}C_xC_z \end{array}\right). \tag{48}$$

Where

$$A = \left(\frac{n-p-q-r}{n-q-r}\right), \quad B = \left(\frac{n-p-q-r}{n-p-r}\right), \quad F = \left(\frac{n-p-q-r}{n-p-q}\right)$$

$$\lambda_1 = \left(\frac{p}{n-q-r}\right), \qquad \lambda_2 = \left(\frac{q}{n-p-r}\right), \qquad \lambda_3 = \left(\frac{r}{n-p-q}\right)$$

$$f_{p+q+r} = \left(\frac{1}{n-p-q-r}\right) - \frac{1}{N}, \qquad f_{p+q} = \left(\frac{1}{n-p-q}\right) - \frac{1}{N}$$

$$f_{p+r}=\left(\frac{1}{n-p-r}\right)-\frac{1}{N},\qquad f_{q+r}=\left(\frac{1}{n-q-r}\right)-\frac{1}{N}$$

Comparison between Proposed and Revised Ratio Estimators

Comparison of proposed exponential type ratio estimators and revised estimators.

For comparison of $\bar{y}_{r1}$ and $\bar{y}_{er1}$, $\bar{y}_{er1}$ is efficient than $\bar{y}_{r1}$, if

$$MSE(\bar{y}_{r1})-MSE(\bar{y}_{er1})>0$$

$$\begin{pmatrix}\frac{3}{4}f_{p+q+r}C_x^2+\frac{3}{4}f_{p+q+r}C_z^2-f_{p+q}\rho_{xy}C_xC_y\\ -f_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}f_{p+r}\rho_{xz}C_xC_z\end{pmatrix}>0$$

For comparison of $\bar{y}_{r2}$ and $\bar{y}_{er2}$, $\bar{y}_{er2}$ is efficient than $\bar{y}_{r2}$, if

$$MSE(\bar{y}_{r2})-MSE(\bar{y}_{er2})>0$$

$$\begin{pmatrix}\frac{3}{4}A^2f_{p+q+r}C_x^2+\frac{3}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2+\frac{3}{4}f_{p+q+r}C_z^2-Af_{p+q}\rho_{xy}C_xC_y\\ -f_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}Af_{p+r}\rho_{xz}C_xC_z\end{pmatrix}>0$$

For comparison of $\bar{y}_{r3}$ and $\bar{y}_{er3}$, $\bar{y}_{er3}$ is efficient than $\bar{y}_{r3}$, if

$$MSE(\bar{y}_{r3})-MSE(\bar{y}_{er3})>0$$

$$\begin{pmatrix}\frac{3}{4}F^2f_{p+q+r}C_z^2+\frac{3}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2+\frac{3}{4}f_{p+q+r}C_x^2-f_{p+q}\rho_{xy}C_xC_y\\ -Ff_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}Ff_{p+r}\rho_{xz}C_xC_z\end{pmatrix}>0$$

For comparison of $\bar{y}_{r4}$ and $\bar{y}_{er4}$, $\bar{y}_{er4}$ is efficient than $\bar{y}_{r4}$, if

$$MSE(\bar{y}_{r4})-MSE(\bar{y}_{er4})>0$$

$$\left(\begin{array}{l}\frac{3}{4}F^2 f_{p+q+r}C_z^2+\frac{3}{4}f_{p+q+r}C_x^2-Bf_{p+q}\rho_{xy}C_xC_y\\ -Bf_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}f_{p+r}\rho_{xz}C_xC_z\end{array}\right)>0$$

For comparison of $\bar{y}_{r5}$ and $\bar{y}_{er5}$, $\bar{y}_{er5}$ is efficient than $\bar{y}_{r5}$, if

$$MSE(\bar{y}_{r5})-MSE(\bar{y}_{er5})>0$$

$$\left(\begin{array}{l}\frac{3}{4}A^2 f_{p+q+r}C_x^2+\frac{3}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2+\frac{3}{4}F^2 f_{p+q+r}C_z^2+\frac{3}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2\\ -Af_{p+q}\rho_{xy}C_xC_y-Ff_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}AFf_{p+r}\rho_{xz}C_xC_z\end{array}\right)>0$$

For comparison of $\bar{y}_{r6}$ and $\bar{y}_{er6}$, $\bar{y}_{er6}$ is efficient than $\bar{y}_{r6}$, if

$$MSE(\bar{y}_{r6})-MSE(\bar{y}_{er6})>0$$

$$\left(\begin{array}{l}\frac{3}{4}A^2 f_{p+q+r}C_x^2+\frac{3}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2+\frac{3}{4}F^2 f_{p+q+r}C_z^2\\ -ABf_{p+q}\rho_{xy}C_xC_y-Bf_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}Af_{p+r}\rho_{xz}C_xC_z\end{array}\right)>0$$

For comparison of $\bar{y}_{r7}$ and $\bar{y}_{er7}$, $\bar{y}_{er7}$ is efficient than $\bar{y}_{r7}$, if

$$MSE(\bar{y}_{r7})-MSE(\bar{y}_{er7})>0$$

$$\left(\begin{array}{l}\frac{3}{4}F^2 f_{p+q+r}C_z^2+\frac{3}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2+\frac{3}{4}F^2 f_{p+q+r}C_x^2\\ -BFf_{p+q}\rho_{xy}C_xC_y-Bf_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}Ff_{p+r}\rho_{xz}C_xC_z\end{array}\right)>0$$

For comparison of $\bar{y}_{r8}$ and $\bar{y}_{er8}$, $\bar{y}_{er8}$ is efficient than $\bar{y}_{r8}$, if

$$MSE(\bar{y}_{r8})-MSE(\bar{y}_{er8})>0$$

$$\left(\begin{array}{l}\frac{3}{4}F^2 f_{p+q+r}C_x^2+\frac{3}{4}\lambda_1^2\left(\frac{1}{p}-\frac{1}{N}\right)C_x^2+\frac{3}{4}F^2 f_{p+q+r}C_z^2\\+\frac{3}{4}\lambda_3^2\left(\frac{1}{r}-\frac{1}{N}\right)C_z^2-AFf_{p+q}\rho_{xy}C_xC_y-BFf_{q+r}\rho_{yz}C_yC_z+\frac{3}{2}AFf_{p+r}\rho_{xz}C_xC_z\end{array}\right)>0$$

Simulation Study

To obtain the mean square error (MSE), of the proposed exponential ratio-type estimators, a simulation study is conducted. Trivariate random observation (X, Y, Z) are generated form a trivariate normal distribution with known correlation coefficients $\rho_{yx}=0.90$, $\rho_{yz}=0.80$ and $\rho_{zx}=0.70$. Using 20,000 simulations, estimates of mean square (MSE) for different revised ratio estimators and exponential ratio-type estimators are computed using simple random sampling without replacement.

While performing a simulation study, we use the following steps in sequence:

1) A sample, say "S" of size n is drawn from trivariate normal distribution using simple random sampling without replacement (SRSWOR).
2) Take the fixed values of missingness rates that is p, q and r.
3) Randomly, we deleted q observation from the set of observations of the study variable and p observations from the set of observation of auxiliary variable X and r observations from the set of observations of the auxiliary variable Z.
4) Identify the sub-samples, S_1, S_2, S_3 and S_4.
5) The values of the estimators $\bar{y}_{r1}$, $\bar{y}_{r2}$, $\bar{y}_{r3}$, $\bar{y}_{r4}$, $\bar{y}_{r5}$, $\bar{y}_{r6}$, $\bar{y}_{r7}$, $\bar{y}_{r8}$, , $\bar{y}_{er1}$, $\bar{y}_{er2}$, $\bar{y}_{er3}$, $\bar{y}_{er4}$, $\bar{y}_{er5}$, $\bar{y}_{er6}$, $\bar{y}_{er7}$, and $\bar{y}_{er8}$, for each (n, p, q, r).
6) Calculate the mean square error of these estimators by 20,000 values that are obtained from 20,000 different simulated samples drawn from the trivariate normal distribution

We have taken the different values of (n, p, q, r) as shown in Table I. in the remaining Tables we have mentioned the MSE of the values of various estimators, considered in this paper obtained for the simulated 20,000 different samples drawn from the trivariate normal distribution on taking various values of sample size and values of missingness rates.

We use the following expression to obtain the MSE of revised ratio estimators and exponential type-ratio estimators.

$$MSE\left(\bar{y}_{ri}\right)=\frac{1}{20000}\sum_{j=1}^{20000}\left(\bar{y}_{ri}j-\bar{Y}\right)^2, \text{ i=1, 2, 3… 8., and}$$

$$MSE\left(\bar{y}_{eri}\right)=\frac{1}{20000}\sum_{j=1}^{20000}\left(\bar{y}_{eri}j-\bar{Y}\right)^2, \text{ i=1, 2, 3… 8}$$

Table 1: MSE of various estimators for sample size n= 18

Missing Rates	$\bar{y}_{r1}$	$\bar{y}_{er1}$	$\bar{y}_{r2}$	$\bar{y}_{er2}$	$\bar{y}_{r3}$	$\bar{y}_{er3}$	$\bar{y}_{r4}$	$\bar{y}_{er4}$
p=3,q=4,r=4	2.396574	0.7393828	1.951486	0.681602	2.268592	0.7426488	2.161716	0.5694147
p=3,q=4,r=5	3.182107	0.9396066	2.412033	0.858222	3.353745	1.0062631	2.753817	0.6734712
p=4,q=3,r=5	3.182107	0.9396066	2.333514	0.843724	2.669765	0.9043809	2.747011	0.7063754
p=4,q=4,r=3	2.396573	0.7393828	1.865966	0.672176	1.545567	0.6171172	2.175548	0.5791479
p=5,q=4,r=4	5.756271	1.1660910	3.485118	1.010267	2.582204	0.9348071	5.186588	0.8145607
p=5,q=5,r=3	5.567432	1.1237691	3.485118	1.010263	2.170051	0.8423768	5.188031	0.7701961

Table 2: MSE of various estimators for some fixed values of p,q and r and sample size 18

Missing Rates	$\bar{y}_{r5}$	$\bar{y}_{er5}$	$\bar{y}_{r6}$	$\bar{y}_{er6}$	$\bar{y}_{r7}$	$\bar{y}_{er7}$	$\bar{y}_{r8}$	$\bar{y}_{er8}$
p=3,q=3, r =4	1.525063	0.6038512	1.415927	0.4793775	.618212	0.5123802	1.321057	0.4912402
p=3,q=4, r =4	1.811023	0.6816815	1.720751	0.5164988	.987264	0.5556879	1.549023	0.5013034
p=3,q=4, r =5	2.518683	0.9174592	2.056697	0.6046364	.861051	0.698419	.078814	0.6227554
p=4,q=3, r =5	1.908653	0.8092523	2.009162	0.6236943	2.298357	0.668385	1.611297	0.5849124
p=4,q=4, r =3	1.179694	0.5580709	1.658365	0.5171799	1.363812	0.4730019	1.018424	0.4199439
p=5,q=4, r =4	1.701742	0.8131791	2.980973	0.6705782	2.152867	0.6287638	1.287084	0.5161752
p=5,q=5, r =3	1.404968	0.7301725	2.950365	0.6308736	1.741203	0.532982	1.031353	0.432979

Table 3: MSE of various estimators for some fixed values of p,q and r and sample size 22

Missing Rates	$\bar{y}_{r1}$	$\bar{y}_{er1}$	$\bar{y}_{r2}$	$\bar{y}_{er2}$	$\bar{y}_{r3}$	$\bar{y}_{er3}$	$\bar{y}_{r4}$	$\bar{y}_{er4}$
p=3,q=3, r =4	0.999108	0.39096	.891152	0.374643	0.9944287	0.3962577	0.9099205	0.3339242
p=3,q=4, r =4	.085283	0.4302739	1.005413	0.4150353	1.0638312	0.4327144	0.9953391	0.3529543
p=3,q=4, r =5	1.380015	0.4949726	1.180753	0.4665746	1.4970909	0.5291145	1.2695542	0.4066787
p=4,q=3, r =5	1.380015	0.4949726	.143156	0.4606994	1.2707411	0.4905026	1.3005861	0.4232593
p=4,q=4, r =3	.085283	0.4302739	.980473	0.4096304	0.8482892	0.3861672	1.0040922	0.3566696
p=5,q=4, r =4	1.370228	.5162657	1.146257	0.4835353	0.9920121	0.4482112	1.2026395	0.4024795
p=5,q=5, r =3	1.370228	0.5162657	1.146257	0.4835353	0.8995361	0.4242284	1.1778943	0.3829085

Table 4: MSE of various estimators for some fixed values of p,q and r and sample size 22

Missing Rates	$\bar{y}_{r5}$	$\bar{y}_{er5}$	$\bar{y}_{r6}$	$\bar{y}_{er6}$	$\bar{y}_{r7}$	$\bar{y}_{er7}$	$\bar{y}_{r8}$	$\bar{y}_{er8}$
p=3,q=3, r =4	0.8905223	0.3792796	0.8162847	0.3188273	0.8994191	0.3340154	0.802313	0.318373
p=3,q=4, r =4	0.9739273	0.4165177	0.9166964	0.3384977	0.9608594	0.3486618	.872982	0.333488
p=3,q=4, r =5	.2750213	0.4979822	1.0850331	0.3807683	1.3500784	0.4277337	1.146436	0.399456
p=4,q=3, r =5	1.0491411	0.4558856	1.0667425	0.3902599	1.1781612	0.4137454	.960956	0.380466
p=4,q=4, r =3	.7497597	0.3658224	0.8934024	0.3356871	0.7810467	0.3173431	.680669	0.297198
p=5,q=4, r =4	0.8212972	0.4193479	0.9956901	0.3727065	0.8629927	0.3468974	.708887	0.321091
p=5,q=5, r =3	0.7532925	0.3979652	0.9679193	0.3525826	0.7763933	0.3153241	0.637463	0.290576

CONCLUSION

The simulation results of Tables 1 and 2 showed that the newly proposed exponential ratio type estimators have minimum MSE as compared to the revised ratio type estimator for sample size 18 and different non response rates. Similarly we compared the result for sample size 22 which are shown in Tables 3 and 4. Therefore the proposed estimators are efficient than all competitor estimators under different missing rates of non-response. So, we can say that our proposed estimator will estimate the mean of the finite population more accurately.

REFERENCES

Diana, G. and Perri, P. F. (2007). Estimation of finite population mean using multiauxiliar information. *Metron*, 99-112.

Grover, L. K. and Kaur, P. (2014). Exponential ratio type estimators of population mean under non response. *Open Journal of Statistics*. 4, 97-110.

Hansen, M. H. and Hurwitz, W. N. (1946). The problems of non-response in sample surveys. *Journal of American. Statistical Association.* 41, 517-529.

Jain, R. K. (1987). Properties of estimators in simple random sampling using auxiliary variable. *Metron*, 265-271.

Khare, B. B. and Srivastawa, S. (1997). Transformed ratio type estimators for the population mean in the presence of non-response. *Communication in Statistics: Theory and Methods*. 26(7), 1779-1791.

Naik, V. D. and Gupta, P. C. (1991). A general class of estimators for estimating population mean using auxiliary information. *Metrika*, 38, 11-17.

Okafor, F. C. and Lee, H. (2000). Double sampling for ratio and regression estimation with sub-sampling the non-respondents. *Survey Methodology*, 26(2), 183-188

Singh, H. P. and Agnihotri, N. (2008). A general procedure of estimating population mean using auxiliary information in sample surveys. *Statistics in Transition*, 9, 71-87.

Singh, H. P. and Kumar, S. (2008). A regression approach to the estimation of the finite population mean in the presence of non-response. *Aust. and N.Z. J. Statist.*, 50(4), 395-408.

Singh, H. P. and Kumar, S. (2010). Improves estimation of population mean under two phase sampling with subsampling the non-respondents. *J. Statist. Plann. and Infer.*, 140, 2536-2550.

Tabasum, R. and Khan, I. A. (2004). Double sampling for ratio estimation with non-response. *J. Ind. Social and Agri. Statist.*, 58, 300-306.

Tabasum, R. and Khan, I. A. (2006). Double sampling for ratio estimator for the population mean in presence of non-response. *Assam Statistical Review*, 20, 73 -83.